Quiz 2 Polymer Physics Fall 2000
10/4/00

In class we evaluated the integral for the positional time correlation function for Brownian motion of a harmonic oscillator,

$$\langle x(t)x(0)\rangle = \int_{-\infty}^{t} dt_1 \int_{-\infty}^{0} dt_2 \exp\left[-k(t - t_1 - t_2)/\zeta\right]\langle g(t_1)g(t_2)\rangle$$

using <g(t)g(t')> = 2Dδ(t-t') and D = kT/ζ. The integral results in <x(t)x(0)> = kTexp(-t/τ)/k_{spr}.

a) **Describe g(t).**

b) The delta function has a value only when its argument is 0, and a value of 0 for all other values of its argument. The integral from -∞ to +∞ of the delta function is one, so

$$\int_{x=-\infty}^{+\infty} f(x)\delta(x - a)dx = f(a)$$

-**First convert** the second integral above to an integral from -∞ to +∞ and **then substitute the delta function** expression for <g(t_1)g(t_2)>.

-**Make an argument** why this conversion is appropriate for a random process such as Brownian motion while it might not generally be appropriate.

c) -**Use the expression** in "b" to reduce the double integral to a single integral and solve the single integral using $\int e^{ax}dx = \frac{1}{a}e^{ax}$.

-**Reduce the** result to the answer given above.

d) For Brownian motion of a harmonic oscillator we obtained the expression:

$$x(t) = \int_{-\infty}^{t} dt'\, e^{-k(t-t')/\zeta} g(t')$$

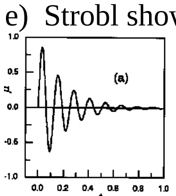
Using the Langevin equation dx/dt = -kx/ζ + g(t) with no inertial term (m d^2x/dt^2). In the next section we will discuss the primary response function μ(t) that can be described by the equation:

$x(t) = \int_{-\infty}^{t} dt'\, \mu(t - t')\psi(t')$ where ψ(t) is an arbitrary time dependent force or field and x(t) is a response or displacement.

-**What is** the primary response function for this damped harmonic oscillator?

-**What is the** force function acting on the damped harmonic oscillator in this case?

e) Strobl shows the primary response function for a harmonic oscillator as:



-**Does Strobl's** damped harmonic oscillator match the primary response function you gave for part "d"? **Explain.**

-**What is** the difference between the Langevin equation used by Strobl and that used in class?

Answers Quiz 2 Polymer Physics

a) g(t) is the velocity of the particle in the absence of the potential field, i.e. the velocity for random Brownian motion in the absence of a harmonic oscillator spring.

$g(t) = V(t) = V_0\exp(-6\pi\eta_s a\ (t_1 - t_2)/m) = V_0\exp(-(t_1 - t_2)/\ \tau_V)$

$\langle g(t)\rangle = 0$

$\langle g(t)g(t')\rangle = 2D\delta(t-t')$

b) Since Brownian motion is time invariant, i.e. you can start at any point in time and obtain the same average parameters, then the integral from -∞ to 0 is the same as the integral from -∞ to ∞. This is because there is only one time where the delta function has a value and this time is in the -∞ to 0 range. Then:

$$\langle x(t)x(0)\rangle = \int_{-\infty}^{t} dt_1 \int_{-\infty}^{\infty} dt_2 \exp\left[-k\left(t - t_1 - t_2\right)/\zeta\right](2D\delta\left(t_1 - t_2\right))$$

c) Using the delta function only has a value at t1 = t2 and the integral at this value is the argument at this value so the integral over t2 becomes the function with t1 substituted for t2.

$$\langle x(t)x(0)\rangle = 2D\int_{-\infty}^{t} dt_1 \exp\left[-k\left(t - 2t_1\right)/\zeta\right] = 2D\exp\left[-kt/\zeta\right]\int_{-\infty}^{t} dt_1 \exp\left[2k\left(t_1\right)/\zeta\right] = \left(D\zeta\ /\ k\right)\exp\left[-kt/\zeta\right]$$

d) $\mu(t) = \frac{1}{\zeta} e^{-kt/\zeta}$

for this case $\zeta g(t)$ is the force function. This is the force associated with random Brownian motion.

e) Strobl's damped harmonic oscillator includes an inertial term, mass times acceleration, in the Langevin equation. Strobls function differs from that in part "d" in that there are inertia driven oscillations. The response function for a harmonic oscillator that includes inertia, i.e. that has particles with finite mass is:

$$\mu(t) = \frac{p}{m\omega_1} e^{-\left(b/2m\right)t} \sin\omega_1 t$$

where p is the momentum transferred by delta function impulse b is the friction factor, m is the particle mass and ω_1 is the natural frequency of resonation for the harmonic oscillator (See p. 851 "Mathematical Methods for Physicists" 3'rd ed by G. Arfken for instance.)